Applying Measurement Uncertainty To Digital Multimeter Calibration

An introductory study of measurement uncertainty and its application to digital multimeter calibration

Teleconference:
US & Canada Toll Free Dial-In Number: 1-(866) 230-5936
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Conference Code: 1010759559

Welcome

Greetings from –

Fluke Corporation
Everett, Washington, USA

We are very pleased to bring you this presentation on measurement uncertainty for DMM Calibration.

Welcome and Thanks!

This presentation is based on Fluke's extensive experience with:

- **Use and design of calibration Instruments**
- **Our experience and understanding of the problems faced when applying measurement uncertainty for both regular and accredited metrology**

Thanks for your time, we hope you find it both valuable and useful.

Presented by

Fluke's Calibration Business Unit

and Jack Somppi

Electrical Calibration Instruments

Product Line Manager

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Web seminar etiquette

- Choice of Audio – VOIP or Teleconference
  - VOIP receives audio only while teleconference is two way sound
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Applying Measurement Uncertainty To Digital Multimeter Calibration

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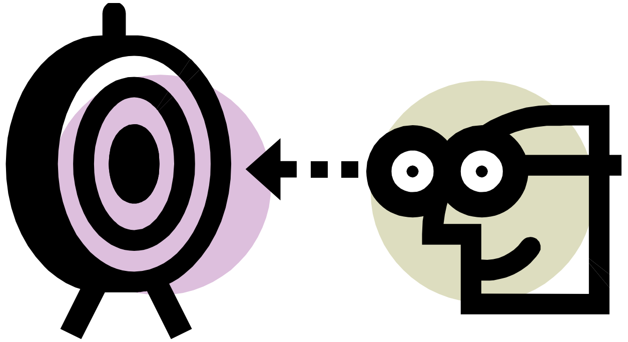
Objectives

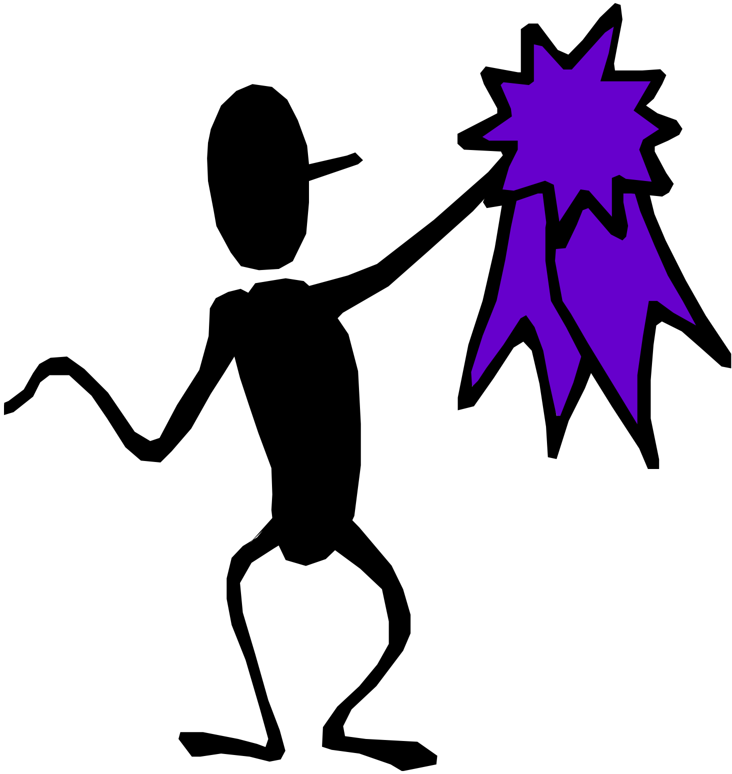


In this session you will -

- **Be introduced to the concept of measurement uncertainty and why it is important**
- **Observe the basic elements that influence measurement uncertainty for DMM calibration applications**
- **Study a simple but detailed example of calculating measurement uncertainty**
- **Consider some benefits of automating measurement uncertainty calculations**
- **Receive a variety of references for further research on this topic**

Benefits

- **Introduce measurement uncertainty to those in calibration/metrology who are not familiar with it**
- **Understand why measurement uncertainty is important for quality metrology**
- **Understand measurement uncertainty with respect to DMM calibration**
- **Appreciate to the benefits of automation**
- **Have technical references for more detailed information**
- **Obtain copies of this presentation via email**



Measurement Uncertainty & Why It Is Important

Facts regarding measurement -

- Can you ever measure the true value of something?
  - **No, there will always be errors**
- How important is this fact?
  - **Very important, as measurement is never complete unless you know how good it is!**
- How is this taken into account in today's calibration & metrology?
  - **By applying & documenting the measurement uncertainty process to the tests being done**

Measurement uncertainty in metrology today…

In The Past

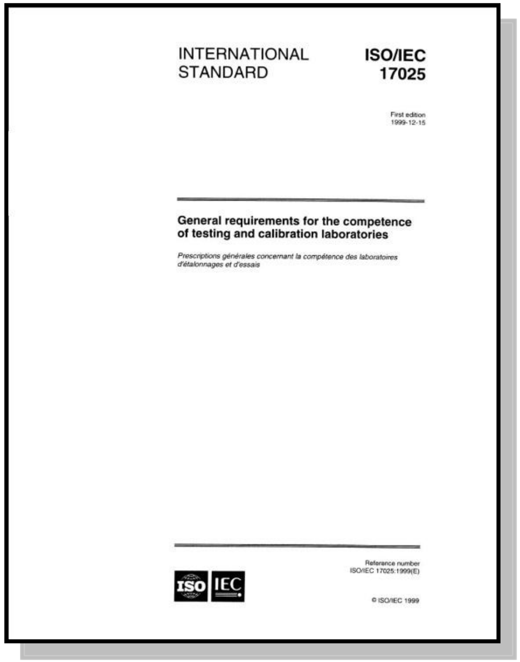
Measurement errors were not rigorously evaluated in all cases. Often in industrial labs, accuracy ratio analysis (referred to as TUR's or TAR's or TSR's) had been frequently used to evaluate the significance of the calibrator's errors on the measurements. Other errors were sometimes ignored.

The Future

Individually analyzed, calculated, & documented measurement uncertainties are more thorough and are required to be considered - as stated in

- **ANSI/ISO/IEC 17025:2005 General Requirements for the Competence of Testing and Calibration Laboratories**

ISO 17025 – about measurement uncertainty…


INTERNATIONAL STANDARD

ISO/IEC 17025

First edition 1999-12-15

General requirements for the competence of testing and calibration laboratories

Prescriptions générales concernant la compétence des laboratoires d'étalonnages et d'essais

Reference number ISO/IEC 17025:1999(E)

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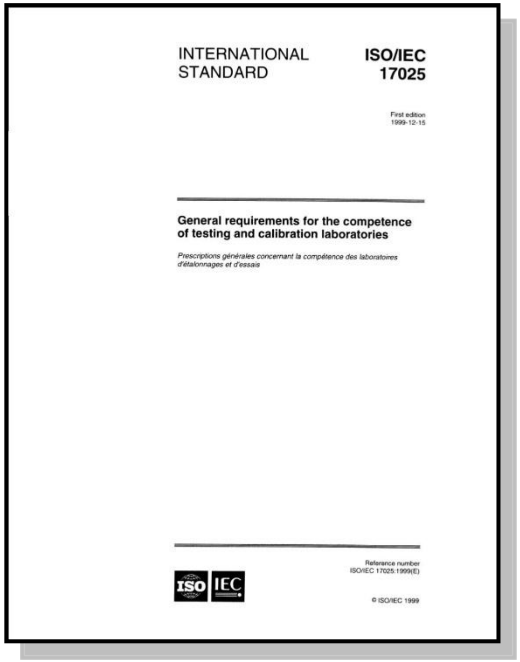
5.4.6 Estimation of uncertainty of measurement

- 5.4.6.1 A calibration laboratory, or a testing laboratory performing its own calibrations, shall have and shall apply a procedure to ***estimate the uncertainty of measurement*** for all calibrations and types of calibrations.

… about the sources of uncertainty…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ISO 17025, Section 5.4.6.3:

- NOTE 1: Sources contributing to the uncertainty include, but are not necessarily limited to,
  - **The reference standards and reference materials used**
  - **Methods and equipment used**
  - **Environmental conditions**
  - **Properties and condition of the item being tested or calibrated**
  - **Operator**

There are many contributors to uncertainty

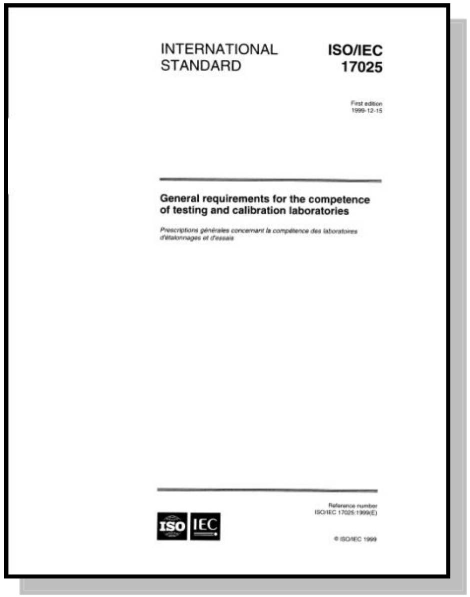
…about calibration certificates…

ISO 17025, Section 5.10.4

Calibration Certificates shall include …
for the interpretation of calibration results

a. The conditions of the test
b. The *uncertainty of measurement* & compliance statements to metrological standards
c. Evidence of traceability

When statements of compliance are made, the uncertainty of measurement shall be taken into account



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This laboratory maintains A2LA accreditation to ISO/IEC 17025 for the specific calibrations listed in Certificate # 2166.01 and meets the relevant requirements of ISO 9001:1994

FLUKE ® Everett Service Center
1420 75th St. SW
Everett, Washington 98203
USA

NQA ISO 9000: 2000 Certified

Calibration Certificate

Description:	CALIBRATOR	**Certificate Number:**	**673712-8430001:1157123340**
Manufacturer:	FLUKE	**Date of Calibration:**	01 September 2006
Model:	5520A/6 120	**Date of Certificate:**	21 September 2006
Serial Number:	8430001	**Date Due:**	
Customer Name:		**Procedure Name:**	FLUKE 5520A: ACAL VER COMBINE
City, State:		**Procedure Revision:**	2.0
Customer Item ID:		**Data Type:**	AS-LEFT
PO Number:		**Temperature:**	23 ± 1.0 °Celsius
RMA Number:	3532046	**Relative Humidity:**	25% ≤ RH ≤ 60%

This calibration certificate may contain data that is not covered by the A2LA Scope of Accreditation. The unaccredited material, where applicable, is indicated by an asterisk (*), or confined to clearly marked sections.

A2LA is a signatory of Mutual Recognition Arrangement for the mutual acceptance of calibration data with the following Laboratory Accreditation systems: Asia Pacific Laboratory Accreditation Cooperation (APLAC), European Cooperation for Accreditation (EA), Inter- American Accreditation Cooperation (IAAC), International Laboratory Accreditation Cooperation (ILAC).

Measurement uncertainties at the time of test are given in the following pages, where applicable. They are calculated in accordance with the method described in NIST TN1297, for a confidence level of 95% using a coverage factor of approximately 2 (K=2).

In the attached measurement measurement results, deviation may be expressed with units, Measured Value (MV) - Nominal Value (NV) or as a proportion of the nominal value ((MV-NV)/NV), expressed without units with a scalar multiplier such as % (0.01), or as a ratio of the units (mW/W, mA/A,mV/V, etc.) Descriptions such as mW/W, mV/V, and others, where used to annotate results or measurement uncertainties, are the preferred replacements for what was historically labeled as "ppm" or parts-per-million and described the results in that column, unless otherwise noted by units symbols.

This certificate applies to only the item identified and shall not be reproduced other than in full, without the specific written approval by Fluke Corporation. Calibration certificates without signatures are not valid. This certificate shall not be used to claim product endorsement by A2LA.

The Data type that could be found in this certificate must be interpreted as:
- As Found - Calibration data collected before the unit is adjusted and/or repaired.
- As Left - Calibration data collected after the unit is adjusted and/or repaired.
- As Found/ As Left - Calibration data collected without any adjustment and/or repair performed.

No statement of compliance with specifications is made or implied on this certificate. However, results are reviewed, if applicable, to establish where any measurement results exceeded the manufacturer's specifications and to communicate results by means of this certificate. Measured values greater than the Manufacturer's specification are indicated by "!".

Comments:

David Chassereau
Metrology Technician

Jorge Martins
Technical Manager

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An example of an accredited calibration certificate –

This laboratory maintains A2LA accreditation to ISO/IEC 17025 for the specific calibrations listed in Certificate # 2166.01 and meets the relevant requirements of ISO 9001:1994

A2LA ACCREDITED

FLUKE ® Everett Service Center
1420 75th St. SW
Everett, Washington 98203
USA

ISO 9001
NQA ISO 9000: 2000 Certified

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Description:	CALIBRATOR	**Certificate Number:**	673712-8430001:1157123340
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Serial Number:	8430001	**Date Due:**	
Customer Name:		**Procedure Name:**	FLUKE 5520A: ACAL VER COMBINE
City, State:		**Procedure Revision:**	2.0
Customer Item ID:		**Data Type:**	AS-LEFT
PO Number:		**Temperature:**	23 ± 1.0 °Celsius
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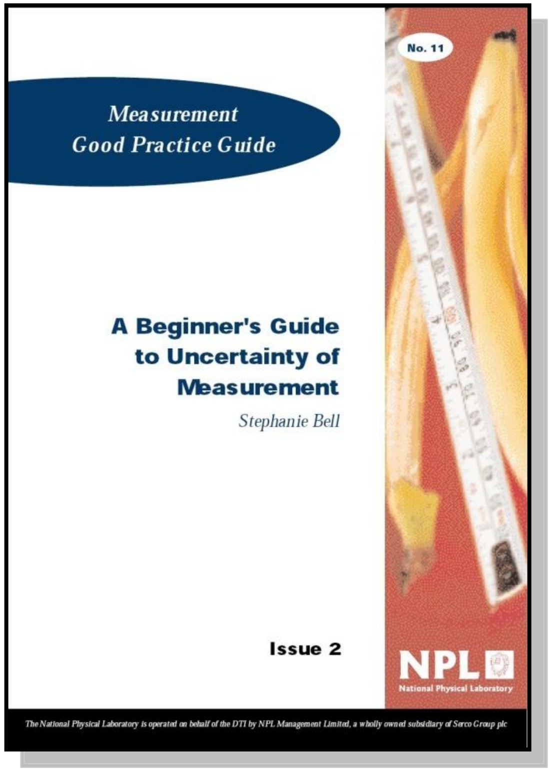
Calibration Results

Function/Range	Nominal Value	Measured Value	Measurement Uncertainty	Manufacturer's Specifications Lower Limit	Manufacturer's Specifications Upper Limit
0.0000 mV	0.0000	0.00014	2.2e-007 V	-0.00100	0.00100
300.000 mV	300.000	299.9990	1.2e-006 V	299.9945	300.0055
-300.000 mV	-300.000	-299.9987	1.2e-006 V	-300.0055	-299.9945

To summarize the importance of measurement uncertainty….

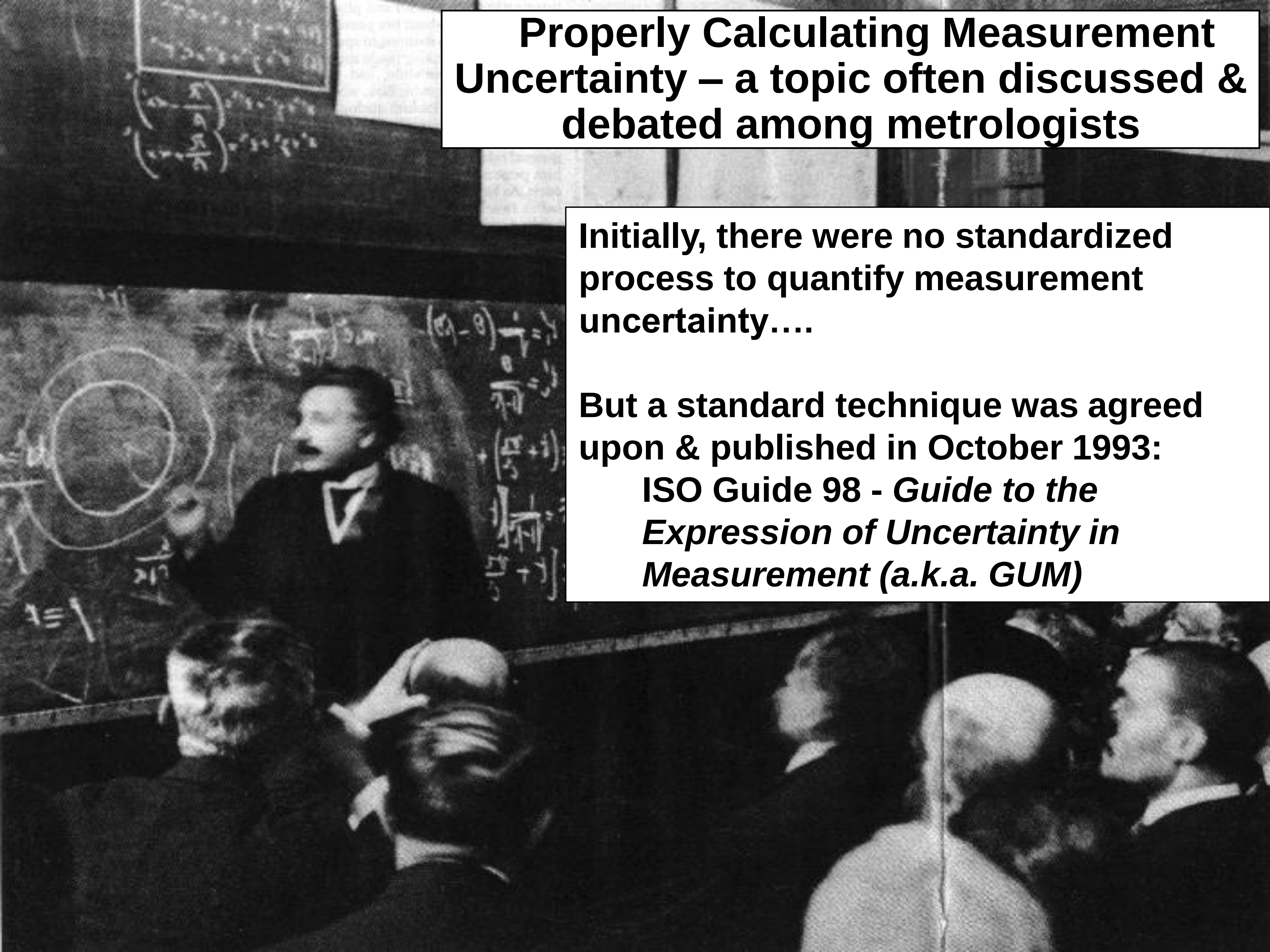
From the NPL UK - "A Beginner's Guide to Uncertainty of Measurement"

- Uncertainty of a measurement tells us something about its quality
- Uncertainty of measurement is the doubt that exists about the results of any measurement
- For every measurement – even the most careful – there is always a margin of doubt
- You need to know the uncertainty before you can decide whether the tolerance is met



“How is this Measurement Uncertainty obtained?”

Properly Calculating Measurement Uncertainty – a topic often discussed & debated among metrologists



Initially, there were no standardized process to quantify measurement uncertainty….

But a standard technique was agreed upon & published in October 1993:

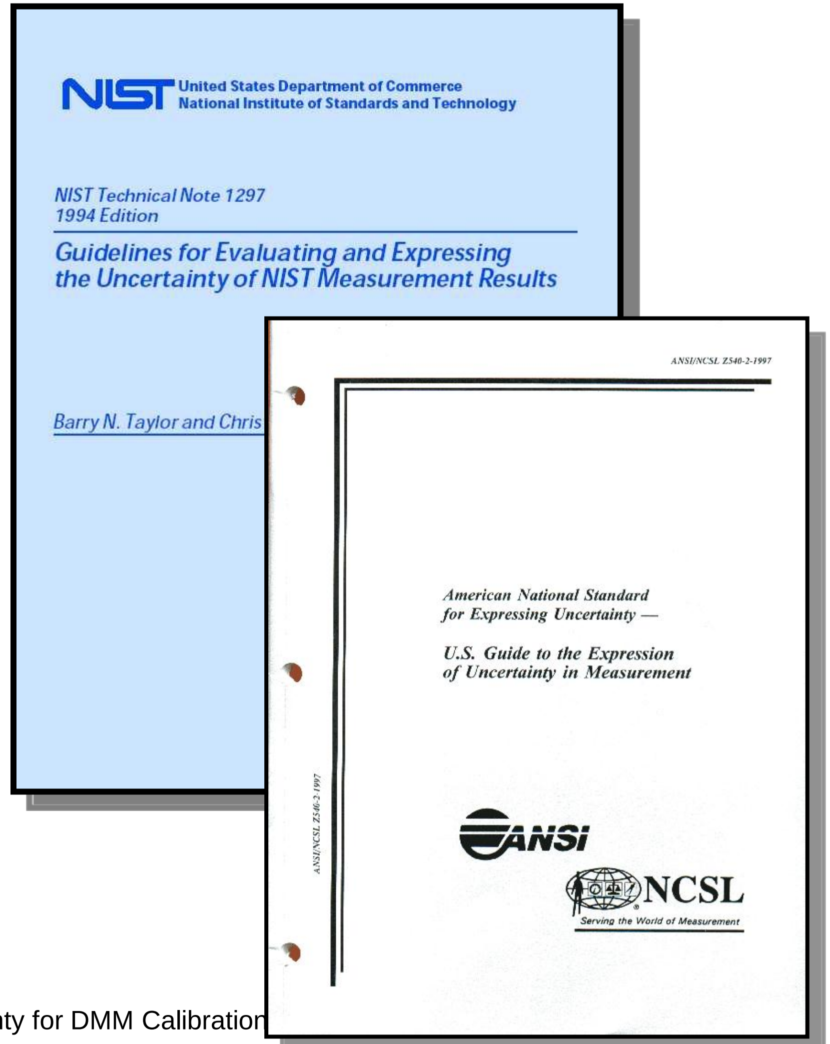
> **ISO Guide 98 - *Guide to the Expression of Uncertainty in Measurement (a.k.a. GUM)***

Recommendation:
Refer to the GUMs -

In the USA, refer to one of the Guides relating to expressing of Uncertainty in Measurement

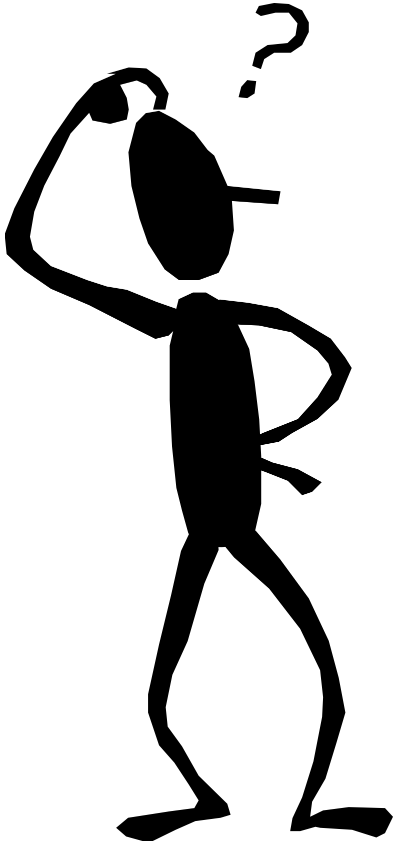
ANSI/NCSL Z540.2-1997 (R2002) U.S. Guide to Expression of Uncertainty in Measurement
http://www.ncsli.org and find it in the store under NCSLI publications

NIST Technical Note 1297
http://www.physics.nist.gov/Pubs/guidelines/contents.html



Internationally, many metrology organizations publish similar GUMs

Questions?
- about measurement uncertainty
or why it is important



Measurement Uncertainty & Calibrating DMMs

A study of applying the GUM to DMM calibration

First – lets look at the concept

Our initial look –

- Consider verifying a precision digital multimeter
- With a hypothetical study of verifying the DMM's measurement performance at 100 millivolts DC
- Let's briefly look at what measurement uncertainty could be in this case

Some sources of measurement "doubt" when verifying a DMM

- The most obvious & significant sources of doubt:
  - **Inaccuracy of the calibrator's output value**
    - **100.0000 mV might actually be 100.0000 mV± .0030 mV**
  - **Repeatability or randomness in measurement values from the DMM**
    - **100.0003 mV, 99.9995 mV, 100.0010 mV, etc.**
  - **Resolution or sensitivity limits on the DMM**
    - **It's value is ±½ the least significant digit,**
    - **in this example it represents ±0.05 µV**
- Many other factors that could also contribute to uncertainty:
  - **ambient temperature effects, thermal emfs, noise, loading, power line conditions, etc.**
- Consider all factors and include if they significantly contribute to measurement uncertainty

The GUMs classify two types of measurement uncertainty

- Type A uncertainty – errors that can be statistically evaluated from the set of measurement data (Often considered as random uncertainty)
  - **For example: Repeatability of the measurement (influenced by dmm characteristics, signal stability, jitter, noise, etc.)**
- Type B uncertainties – estimates of errors influencing the measurement that are not directly observed from the measurement data (Often considered as systematic uncertainty)
  - **Errors of the calibrating standards (performance specifications for accuracy changes over time and other conditions)**
  - **Inherent limitations of the unit being tested (DMM resolution limitations)**

FLUKE

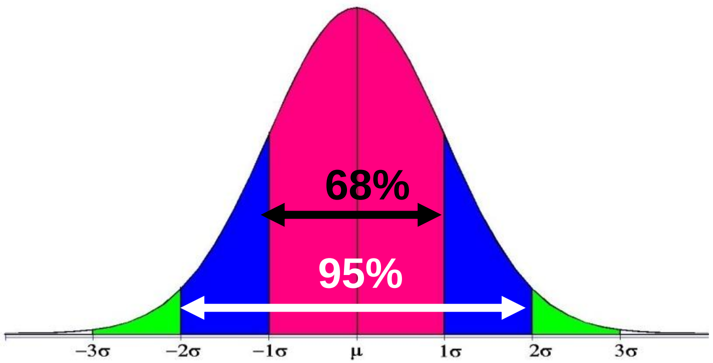
Combining all the uncertainties

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2}$$

- To quantify uncertainty, the various sources of uncertainty need to be quantified, evaluated, & combined
- Calculate a combined estimate of all the individual A and B types of uncertainties
- This combined uncertainty u_c is:
  - **a basic estimate (representing one statistical standard deviation)**
  - **usually the RSS of all individual uncertainties**

(Combining uncertainties using such an RSS technique applies to uncertainties with standard relationships and are independent)

The expanded uncertainty



- As mentioned, calculations for u_c pertain to ± one standard deviation of measurement uncertainties (covering 68% of the population of measurements)
- Usually it is desired to express uncertainty for a larger population or condition, say 95% or 99%.
- Expanding the calculated uncertainty through scaling estimates an uncertainty that covers this larger population - U_m.

$$U_m = k u_c$$

- A coverage factor, k, (often equal to 2), would indicate a 95% confidence.

Now, returning to the … statement of uncertainty

- ... A measurement is complete only when accompanied by a statement of the uncertainty of the estimate. For example:

$$V_{DMM} = 100.0051 mV \pm 0.0004\ mV$$

- In this case, ± 0.0004 mV would be the resulting value of U_m, calculated as shown below:

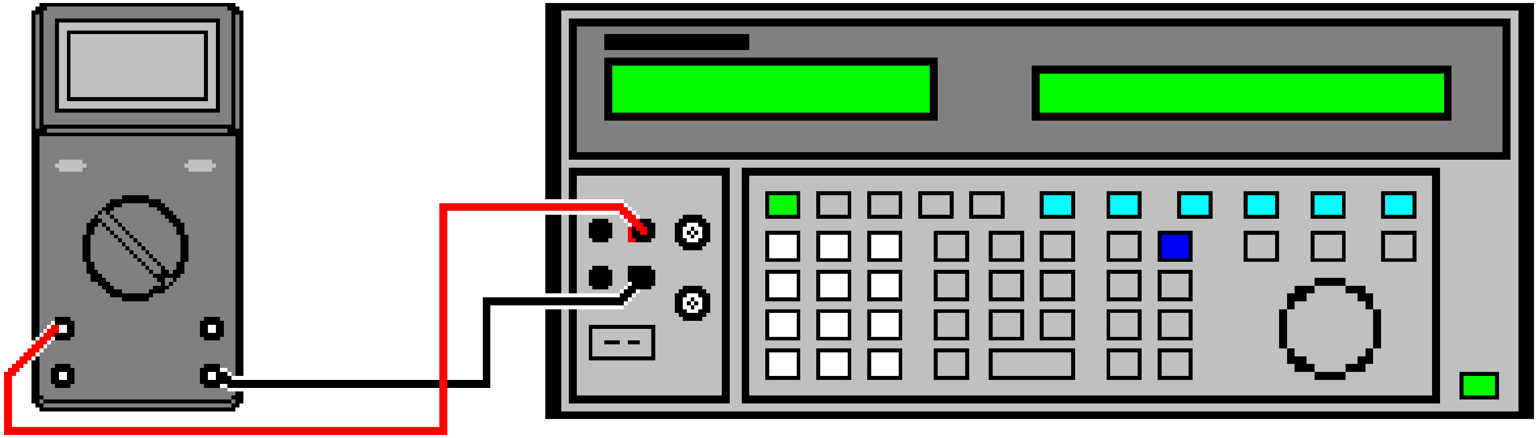
$$0.0004mV = U_m = ku_c$$

$$= k\sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2}$$

That describes the general process – are we okay so far?

Next, a different and more detailed example…

Examine the use of a Fluke 5500A to verify a 3.5 digit DMM at 10 Amps of Alternating Current at 50 Hz



The "A" portion…

- Type A uncertainty is determined by the statistical analysis of a series of observations (measurements).
- Type A uncertainties includes effects from:
  - **Variations of multiple repeated readings from the UUT**
  - **Effects of the system noise**
  - **Noise and short term variation of the standard**
- Now let's examine the basic statistics …

Measured value: the average of a series of measurements

Measurement	Value
1	10.07
2	10.02
3	10.01
4	10.06
5	10.04
Average	10.04

$$I_{avg} = 10.04A$$

- An average of multiple measurements is a better estimate of the true value than any individual value
- As a rule of thumb, taking between 4 & 10 measurements are sufficient.
- Uncertainty improvements for more than 10 have diminishing results
- In our example, 5 readings are sufficient. Any improved uncertainties for more readings are not significant versus required measurement tolerances (a typical DMM specification for this example test is ~ ±2.5%).

Calculating the uncertainty due to measurement repeatability

Measurement	Value	Deviation from Average
x_1	10.07	+0.03
x_2	10.02	-0.02
x_3	10.01	-0.03
x_4	10.06	+0.02
x_5	10.04	0.00

Experimental Standard Deviation

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{(n-1)}}$$

Experimental Standard Deviation of the Mean

$$u_1 = \frac{s}{\sqrt{n}}$$

- The uncertainty is statistically analyzed from the measurement data series

u_1 – for a normally distributed population, the best estimate of uncertainty is the experimental standard deviation of the mean

NOTE: In the unusual case where

1. **the calibrating standard is extremely accurate & stable, and**
2. **the repeated test measurement values are unchanged (or even with only a ± one digit change)**

Then this uncertainty can be considered as non significant

- **One measurement value would be sufficient**
- **The type B resolution uncertainty is adequate**

The estimated standard deviation

Measurement	Value	Deviation from Average
x_1	10.07	+0.03
x_2	10.02	-0.02
x_3	10.01	-0.03
x_4	10.06	+0.02
x_5	10.04	0.00
$\overline{x}$ (Average)	10.04	
s (Estimated Std. Dev.)		0.02549

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{(n-1)}} = 25.5 \text{ mA}$$

u_1 – estimated standard uncertainty

Calculate the Standard Deviation of the Mean

$$u_1 = \frac{s}{\sqrt{n}} = \frac{25.5mA}{\sqrt{5}} = 11.4mA$$

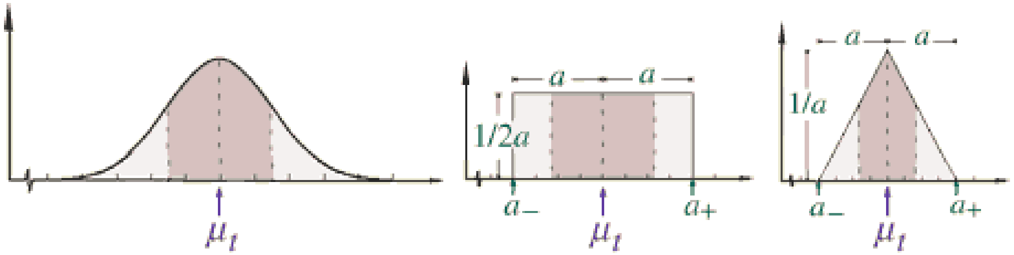
Plus there are some other important characteristics to consider:

- **Probability Distribution = Normal**
- **Sensitivity Coefficient = 1**
- **Degrees of Freedom = 4**

What are these?

Statistical terms & concepts

- Probability Distribution: "the scatter of the values"
  - **Normal or Gaussian**
  - **Rectangular or Uniform**
  - **Triangular, U or bi-modal, …**



- Degrees of Freedom: "how many"
  - **A value related to the amount of information that was employed in making the estimate.**
  - **Usually equals the sample size minus one (n-1) for type A uncertainties, and is often considered infinite (∞) for parameters such as manufacturer specifications**
- Sensitivity Coefficient: "how influential"
  - **Change in measurement response divided by the corresponding change in stimulus (usually a value of 1 in the case we are considering)**

For more information, see technical references on statistics

u_1 – estimated standard uncertainty

Calculate the Standard Deviation of the Mean

$$u_1 = \frac{s}{\sqrt{n}} = \frac{25.5mA}{\sqrt{5}} = 11.4mA$$

- **Probability Distribution = Normal** — Grouped around a value
- **Sensitivity Coefficient = 1** — Direct influence on response
- **Degrees of Freedom = 4** — Based on 5 independent measurements

The ”B” type of uncertainties …

All the other uncertainties that cannot be determined statistically during the measurement process, such as -

- **Calibrator inaccuracy or error**
- **Measurement errors due to limitations of the DMM’s resolution**
- **lead effects, thermal emfs, loading, etc.**

- Estimates here are based on scientific judgment using all relevant information
- Numerically, these are expressed as one standard deviation estimates for each different uncertainty

u_2 - uncertainty due to the calibrator inaccuracy

u_2 is the ±1 sigma estimate of the calibrator error,

- (estimates a ±1 standard deviation coverage of the errors - for 68% of all possible values),
- based on the specifications for performance at the specific test setting
  - **Start with the manufacturer's recommended specifications at the test point**
  - **Adjust as required for any appropriate factors such as legal traceability limitations, improvements for output characterizations, etc.**
  - **Convert to a ± one sigma confidence interval basis**

Refer to the calibrator specifications

5500A
Operator Manual

1-20. AC Current (Sinewave) Specifications

Ranges	Frequency	Absolute Uncertainty, tcal ± 5°C ± (% of output + µA)				Resolution	Compliance Voltage	Maximum Inductive Load
		90 days		1 year				
2.2 to 11 A	45 to 65 Hz	0.05%	2000 µA	0.06%	2000 µA	100 µA	2.8 to 1.25 V rms [3]	200 µH, 45 to 65 Hz 1 µH, 65 Hz to 1 kHz
	65 to 500 Hz	0.08%	2000 µA	0.10%	2000 µA			
	500 Hz to 1 kHz	0.25%	2000 µA	0.33%	2000 µA			

- For this example, assume it is a certified calibrator that is routinely calibrated every year.
- The ***absolute uncertainty specifications*** for 10 Amps, 50 Hz:

  0.06% of output plus 2000 µAmps

Calculating u_2

- Step 1: Calculate the maximum instrument error per manufacturer's specifications at the point of test

5500A – 1 year specs @10 A, 50 Hz

±(0.06% of 10 A + 2000 µA)

is calculated to be:

±(6 mA + 2 mA) = ±8 mA

Calculating u_2

- Step 2: Convert the specified error to an error value that covers ±one standard deviation (or a ±1 sigma confidence interval)

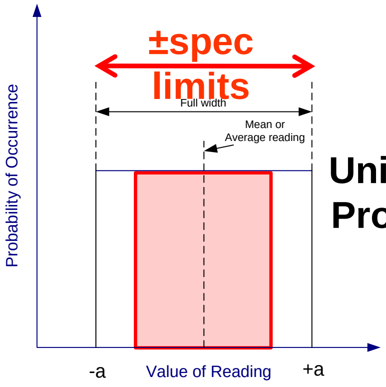
  - **If no other information is provided by the manufacturer, assume a rectangular distribution**

    $$\pm 1\sigma = \pm spec / (\sqrt{3})$$

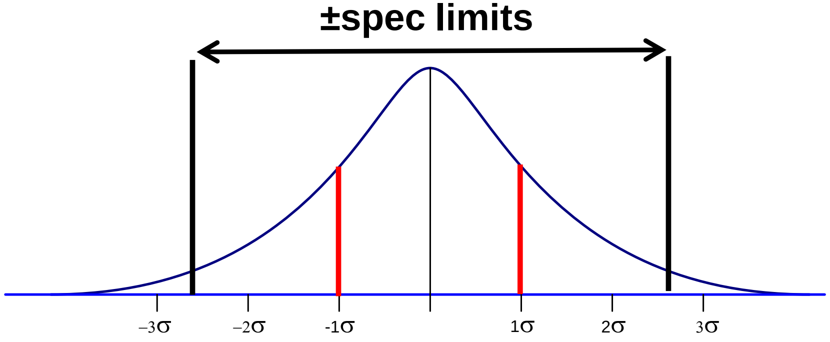
  - **If manufacturer specifies a different distribution, such as a normal distribution, then calculate as appropriate.**

    For example with a normal distribution at 99%

    $$\pm 1\sigma = \pm spec / (2.58)$$



Uniform or Rectangular Probability Distribution



Normal Probability Distribution

Fluke's 5500A specifications

5500A
Operator Manual

1-20. AC Current (Sinewave) Specifications

Ranges	Frequency	Absolute Uncertainty, tcal ± 5 °C ± (% of output + µA)				Re
		90 days		1 year		
2.2 to 11 A	45 to 65 Hz	0.05%	2000 µA	0.06%	2000 µA	
	65 to 500 Hz	0.08%	2000 µA	0.10%	2000 µA	
	500 Hz to 1 kHz	0.25%	2000 µA	0.33%	2000 µA	

1-15. General Specifications

Warmup Time	Twice the time since last w
Settling Time	Less than 5 seconds for a
Standard Interfaces	IEEE-488 (GPIB), RS-232
Temperature Performance	• Operating: 0 °C to 50 °C
Weight (without options)	90-day specification (or 1- 5500A Calibrator, 22 kg (
Absolute Uncertainty Definition	The 5500A specifications load regulation, and the tr You do not need to add ar the temperature range ind
Specification Confidence Interval	99 %

The manufacturer's specs document that specifications are based on a normally distributed, 99% confidence interval

Calculating u_2

- The value of u_2 is the ±1 sigma calibrator spec:

5500A – 1 year specs @10 A, 50 Hz

With a spec of ±8 mA at 99% confidence

divide by 2.58 to convert to a ±1 sigma spec

u_2 **= 8 mA / 2.58 mA = <u>3.1 mA at ±1 std. dev.</u>**

This u_2 value should be ***<u>smaller</u>*** than the published spec!

Summary of u_2 –

u_2 is the ±1 sigma estimate of calibrator specification uncertainty

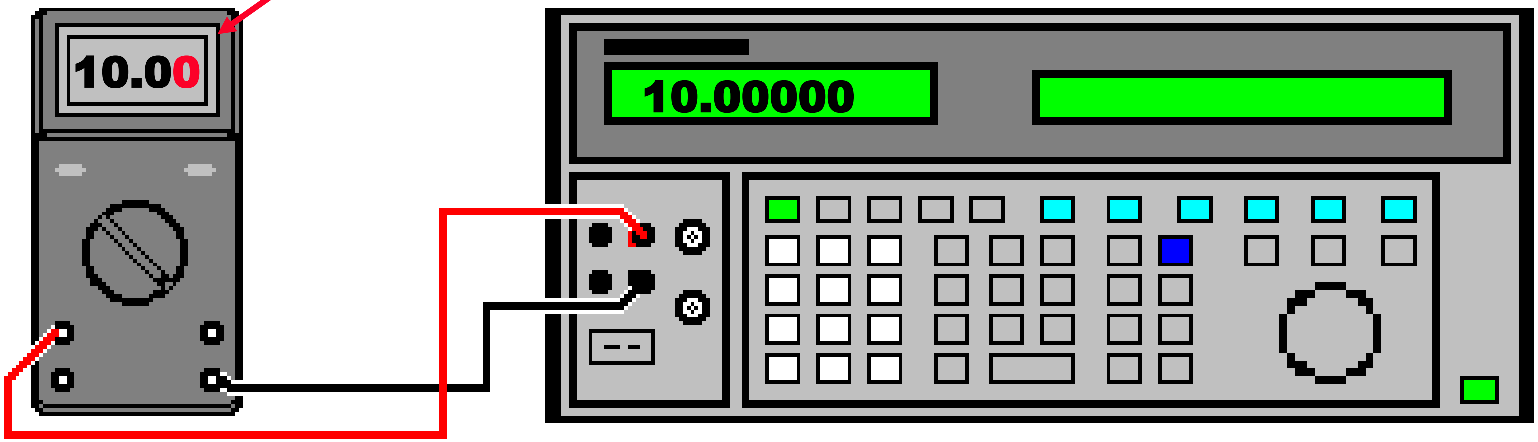
$$u_2 = 3.1mA$$

- **Probability Distribution = Normal – as stated in the manufacturer's information**
- **Sensitivity Coefficient = 1**
- **Degrees of Freedom =** ∞

u_3 - uncertainty due to UUT measurement limitations

- Measurements include error due to resolution limits of the UUT - considered as one half of the LSD
- The LSD of resolution for this UUT measuring 10 Amps is 10 mA

LSD (least significant digit)



Calculating u_3

The formula for u_3 is:

$$u_3 = \frac{1}{2} \times \text{LSD} \Big/ \sqrt{3}$$

Calculates the standard uncertainty related to one LSD

With an LSD of 10 mA -

u_3 = 2.9 mA at a ±1 std. dev.

Summary of u_3 –

u_3 is the ±1 sigma estimate of dmm LSD resolution uncertainty

$$u_3 = 2.9mA$$

- **Probability Distribution = Rectangular**
- **Sensitivity Coefficient = 1**
- **Degrees of Freedom =** ∞

This completes the “B” portion…

u_2 = **3.1 mA at ±1 standard deviation**

u_3 = **2.9 mA at ±1 standard deviation**

- **There are no other “B” uncertainties which are significant for this particular test**

(Note: It is often good to identify and document the other possible uncertainties deemed insignificant.)

Combining all uncertainties …

Standard Combined Uncertainty $= u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2}$

12.16 mA $= \sqrt{11.4^2 + 3.1^2 + 2.9^2}$

A One Standard Deviation Estimate Of Combined Uncertainty

Overall uncertainty budget

Source of Uncertainty	Type	Ui	Uncertainty Value (Amps)	Sensitivity Coefficient	Probability Distribution	Coverage Factor	Standard Uncertainty (Amps)	Degrees of Freedom
Repeatability	A	u_1	**11.4×10^{-3}**	**1**	**Normal**	1	**11.4×10^{-3}**	**4**
Calibrator	B	u_2	**8×10^{-3}**	**1**	**Normal**	2.58	**3.1×10^{-3}**	∞
Resolution	B	u_3	**5×10^{-3}**	**1**	**Rectangular**	$\sqrt{3}$	**2.9×10^{-3}**	∞
Current Measurement	**Combined**	u_C	**-**	**-**	**Assumed Normal**	**-**	**12.16×10^{-3}**	**5.2**

How do you calculate the overall effective Degrees of Freedom?

Welch-Satterthwaite formula

- ν_{eff} is the overall effective degrees of freedom for the combined uncertainty (u_c).
- The formula considers each uncertainty, each sensitivity coefficient and each uncertainty's specific value for degrees of freedom to calculate ν_{eff}

$$\nu_{eff} = \frac{u_c^4(y)}{\left(\sum_{i=1}^{N} \frac{c_i^4 u^4(x_i)}{\nu_i} \right)}$$

Welch-Satterthwaite formula in our example case

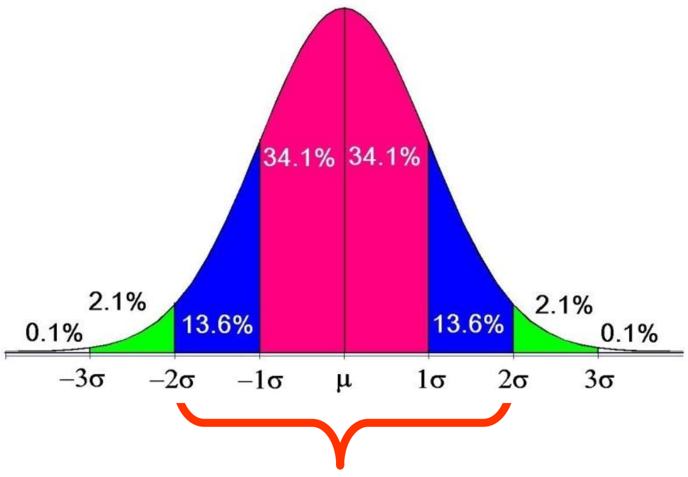
$$v_{eff} = \frac{u_c^4(y)}{\frac{c_1^4 u_1^4(x_1)}{v_1} + \frac{c_2^4 u_2^4(x_2)}{v_2} + \frac{c_3^4 u_3^4(x_3)}{v_3}}$$

$$v_{eff} = \frac{(12.16\times10^{-3})^4}{\frac{1^4\times(11.4\times10^{-3})^4}{4} + \frac{1^4\times(3.1\times10^{-3})^4}{\infty} + \frac{1^4\times(2.9\times10^{-3})^4}{\infty}} = 5.2$$

Our effective degrees of freedom considering all our uncertainties

Calculating the expanded uncertainty

$$U_m = k u_c$$



k is the **coverage factor**

Level of Confidence (percent)	Coverage factor k
68.27%	1
90%	1.645
95%	1.960
95.45%	2.0
99%	2.576
99.73%	3

- **How confident should you be with your measurement results? (68%, 95%, 99%....)**
- **95% confidence is commonly accepted as appropriate.**
- **U_m expresses the uncertainty, expanded from a single standard deviation of 68%, to uncertainty value with a higher confidence.**
- **For a large population with a normal distribution, 95% coverage is calculated by k with a value of 1.96 (or sometimes 2 for convenience – giving 95.45%)**

Adjusting *k* for a smaller set of measurements or samples

Degrees of freedom ν	Fraction *p* in percent					
	68.27	90	95	95.45	99	99.73
1	1.84	6.31	12.71	13.97	63.66	235.8
2	1.32	2.92	4.3	4.53	9.92	19.21
3	1.2	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.6	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.9
7	1.08	1.89	2.36	2.43	3.5	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
20	1.03	1.72	2.09	2.13	2.85	3.42
50	1.01	1.68	2.01	2.05	2.68	3.16
100	1.005	1.66	1.984	2.025	2.626	3.077
∞	1	1.645	1.96	2	2.576	3

- Adjusting ***k*** is done using the: students' t distribution table
- A coverage factor adjustment is needed because our data set had a fewer number of values, rather than a larger set (such as 20, 50, or 100)
- The table lists the proper coverage factor for populations with smaller degrees of freedom

For our example with the effective degrees of freedom (V_{eff}) of 5.2, a coverage factor of 2.57 expands u_c to a value with 95% confidence (compared to 1.96 for an infinite set of measurements/samples).

Expanded measurement uncertainty calculation

$$U_m = k u_c$$

$$U_m = 2.57 \times \mathbf{12.16\ mA}$$

$$U_m = \mathbf{31.26\ mA}$$

Our overall uncertainty budget

Source of Uncertainty	Type	Ui	Uncertainty Value (Amps)	Sensitivity Coefficient	Probability Distribution	Coverage Factor	Standard Uncertainty (Amps)	Degrees of Freedom
Repeatability	**A**	u_1	**11.4×10^{-3}**	1	**Normal**	1	**11.4×10^{-3}**	**4**
Calibrator	**B**	u_2	**7×10^{-3}**	1	**Normal**	2.58	**2.7×10^{-3}**	∞
Resolution	**B**	u_3	**5×10^{-3}**	1	**Rectangular**	$\sqrt{3}$	**2.9×10^{-3}**	∞
Current Measurement	**Combined**	u_C	**-**	-	**Assumed Normal**	-	**12.1×10^{-3}**	**5.2**
Current Measurement	**Expanded**	U_m	**31.26×10^{-3}**	**-**	**Assumed Normal**	**2.57**	**-**	**5.2**

Final results -

- The final measurement value including the measurement uncertainty from the series of DMM measurements of the calibrator

$$I = I_{avg} \pm U_m$$

$$\mathbf{I = 10.04 \pm\ 0.031\ Amps}$$

At a level of confidence of 95%

What if more measurements were taken, does that improve the uncertainty?

Increased degrees of freedom

V_{eff} **= 5 10, 20 or 100**

Degrees of freedom ν	Fraction *p* in percent					
	68.27	90	95	95.45	99	99.73
1	1.84	6.31	12.71	13.97	63.66	235.8
2	1.32	2.92	4.3	4.53	9.92	19.21
3	1.2	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.6	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.9
7	1.08	1.89	2.36	2.43	3.5	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
20	1.03	1.72	2.09	2.13	2.85	3.42
50	1.01	1.68	2.01	2.05	2.68	3.16
100	1.005	1.66	1.984	2.025	2.626	3.077
∞	1	1.645	1.96	2	2.576	3

So U_m improves only 7 mA by taking 73 more measurements

Causes marginal improvements in k and in U_m

- 5 measurements, V_{eff} = 5.2
  - **k = 2.57, U_m= 31 mA**
- 9 measurements, V_{eff} = 10.3
  - **k = 2.23, U_m= 27 mA (4 mA better)**
- 17 measurements, V_{eff} = 20.7
  - **k = 2.09, U_m= 25 mA (2 mA better)**
- 78 measurements, V_{eff} = 100.9
  - **k = 1.984, U_m= 24 mA (1 mA better)**

Does improving U_m beyond ±31 mA by taking more measurements have any practical value?

What's the value of increasing V_{eff} from 5 to ?????

The test tolerance is ±250 mA

- 5 measurements, V_{eff} = 5.2
  - **k = 2.57, U_m = 31 mA**

$$I = 10.04 \pm 0.031 \ Amps$$

Degrees of freedom ν	Fraction *p* in percent					
	68.27	90	95	95.45	99	99.73
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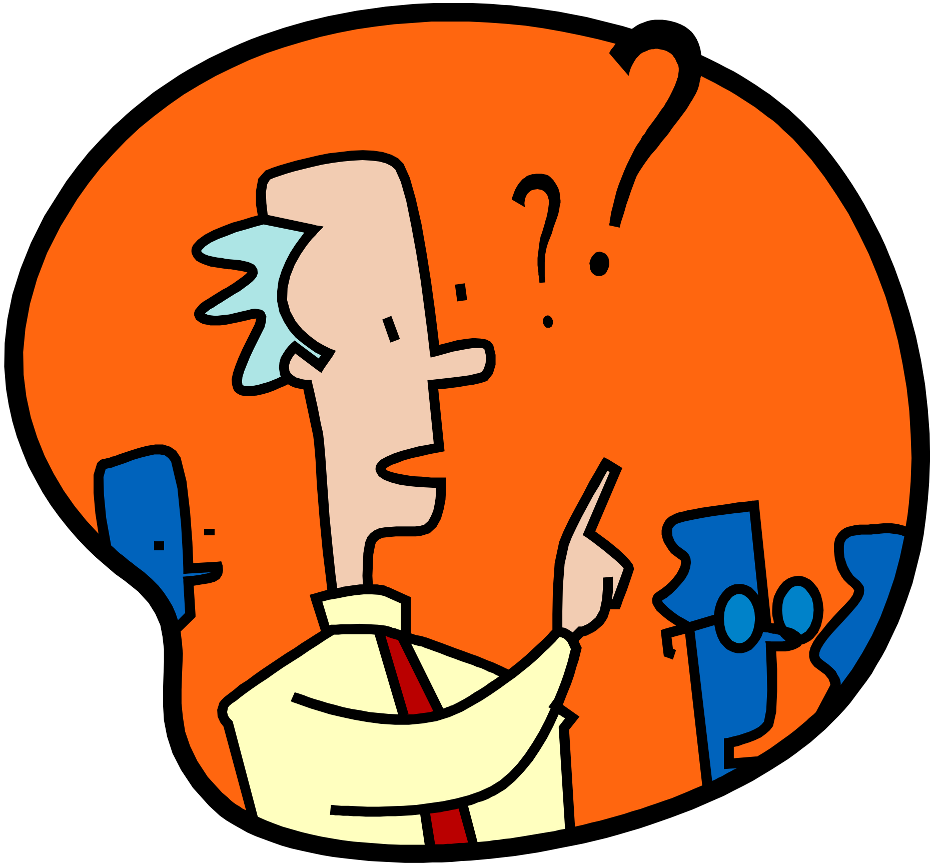
100	1.005	1.66	1.984	2.025	2.626	3.077
∞	1	1.645	1.96	2	2.576	3

- With a U_m = 31mA, the test ratio is already 8:1

(TUR = Test Spec ÷ Total Uncertainty
0.25A ÷ 31mA = 8.06)

So to satisfy a minimum test ratio of 4:1, 5 measurements are more than adequate!

Questions?

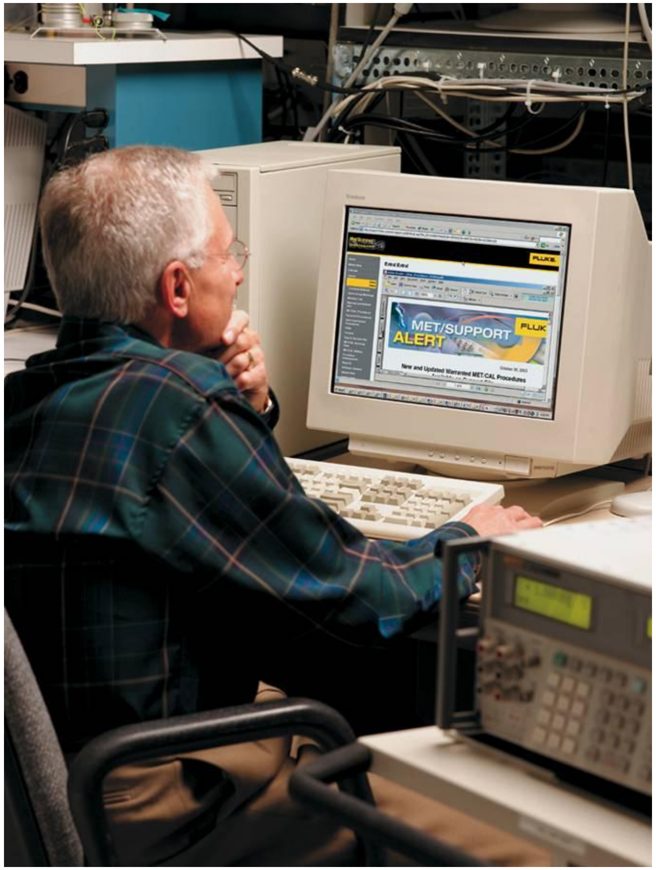


Making The Calculation Of Measurement Uncertainty Simpler

What can you do to automate this?

Automation alternatives

- A custom program designed for a specific requirement
- A custom spreadsheet for analysis
- A commercial metrology based software package such as Fluke's MET/CAL Plus



MET/CAL automates the uncertainty calculations

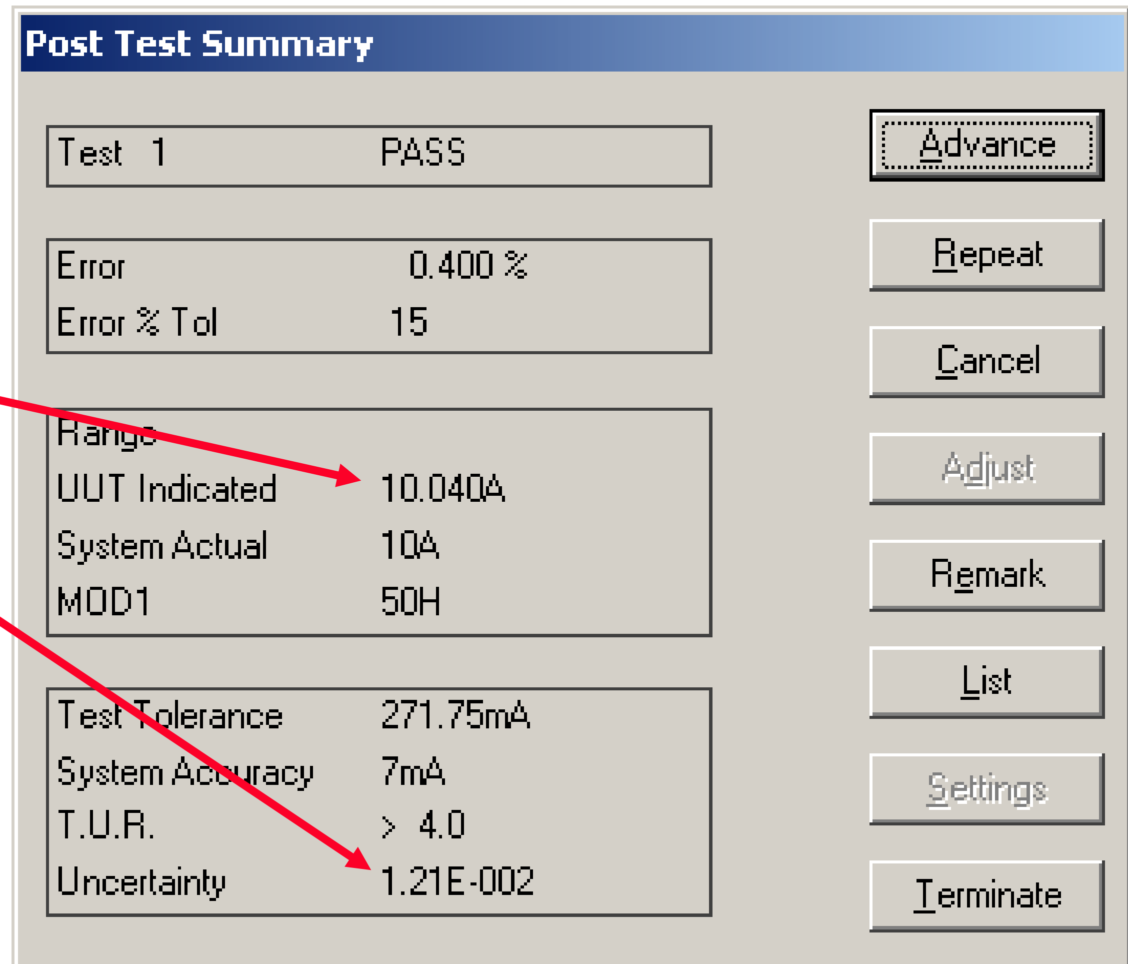
Post test summary of 10.000A @50Hz

Including:

5 reading average

Calculated combined standard uncertainty

How does this work?



MET/CAL manages & analyses the uncertainties

MET/CAL Data for our example

With MET/CAL the user configures:

- Specific statistics used
- Confidence / Coverage
- Number of measurements
- Accuracy of the standard

In the cal or test procedure you also specify test parameters:

- Test point
- UUT resolution

In the test process, MET/CAL provides the uncertainty details (our example is shown to the right)

Details are permanently stored in the data base. They accessible for reports & future analysis.

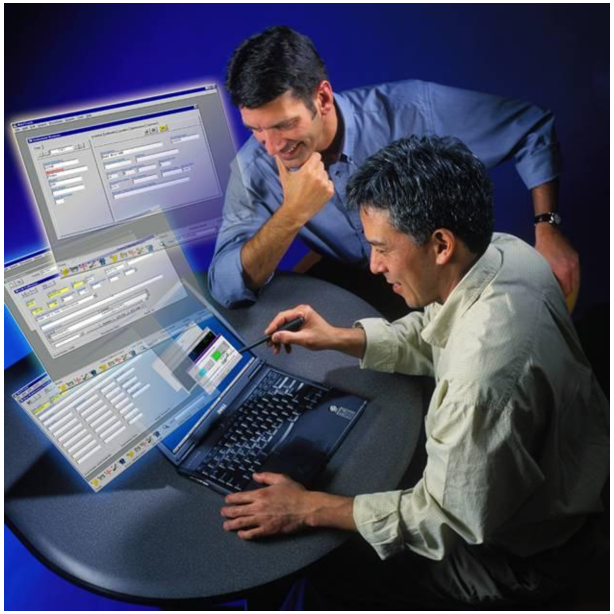
Measurement Details	Number of Measurements	= 5
	Value 1	= 10.07
	Value 2	= 10.01
	Value 3	= 10.02
	Value 4	= 10.04
	Value 5	= 10.06
	UUT Indicated	= 10.04
Repeatability Uncertainty	Standard Deviation	= 0.02549509757
	Standard uncertainty	= 0.01140175425
	Sensitivity Coefficient	= 1
	Degrees of Freedom	= 4
Calibrator Uncertainty	System Actual	= 10
	System Accuracy	= 0.008
	Confidence interval of spec	= 2.58
	1 Sigma Spec	= 0.003126379456
	Sensitivity Coefficient	= 1
	Degrees of Freedom	= 1e+200
Resolution Uncertainty	UUT Resolution	= 0.01
	Resol. Standard Uncertainty.	= 0.002886751346
	Sensitivity Coefficient	= 1
	Degrees of Freedom	= 1e+200
Calculated Total Uncertainty	Combined Std. Uncertainty	= 0.01216490061
	Effective Deg. of Freedom	= 5.186506
	Standard Uncertainty	= 0.01207040471
	Coverage Factor	= 2.567104753
	Expanded Uncertainty	= 0.031263794

"Automation" – some words of wisdom

- Remember, it is always the metrologist's responsibility to insure proper calculation of measurement uncertainty
  - **Every lab has unique characteristics which must be supported**
  - **Configuring the measurement characteristics is also unique**
  - **Defining the specific error budget for the test**
  - **Configuring the specific measurement uncertainty parameters**
- There should be definite information to support answering any auditor's questions
- Keep records of the procedure's measurement design with an uncertainty error budget
- Be able to demonstrate the reasonableness of the test's uncertainties

Benefits of MET/CAL automation

- Automation simplifies a structured calculation process
- Usable for manual, semi automated, or fully automated testing methods
- MET/CAL provides flexibility to customize the calculation process & factors
- MET/CAL’s database stores all the information for future reference
- Report writing flexibility permits properly configured certificates and data summaries
- Lets the technical staff concentrate on the test quality rather than the rote mathematical & statistical processes



Automation questions?

Conclusion & Review – What have we done?

- Topics
  - **Measurement uncertainty & why it is important**
  - **How measurement uncertainty obtained**
  - **Examples on measurement uncertainty & calibrating DMMs**
  - **Benefits of automating**
- Measurement Uncertainty is becoming an essential consideration in all metrology & calibration measurements
- Measurement results are considered incomplete without a quoted uncertainty
- Calculations usually require a statistical process on multiple measurements for each test
- Automation can be a valuable support for measurement uncertainty calculations

Where to go from here?

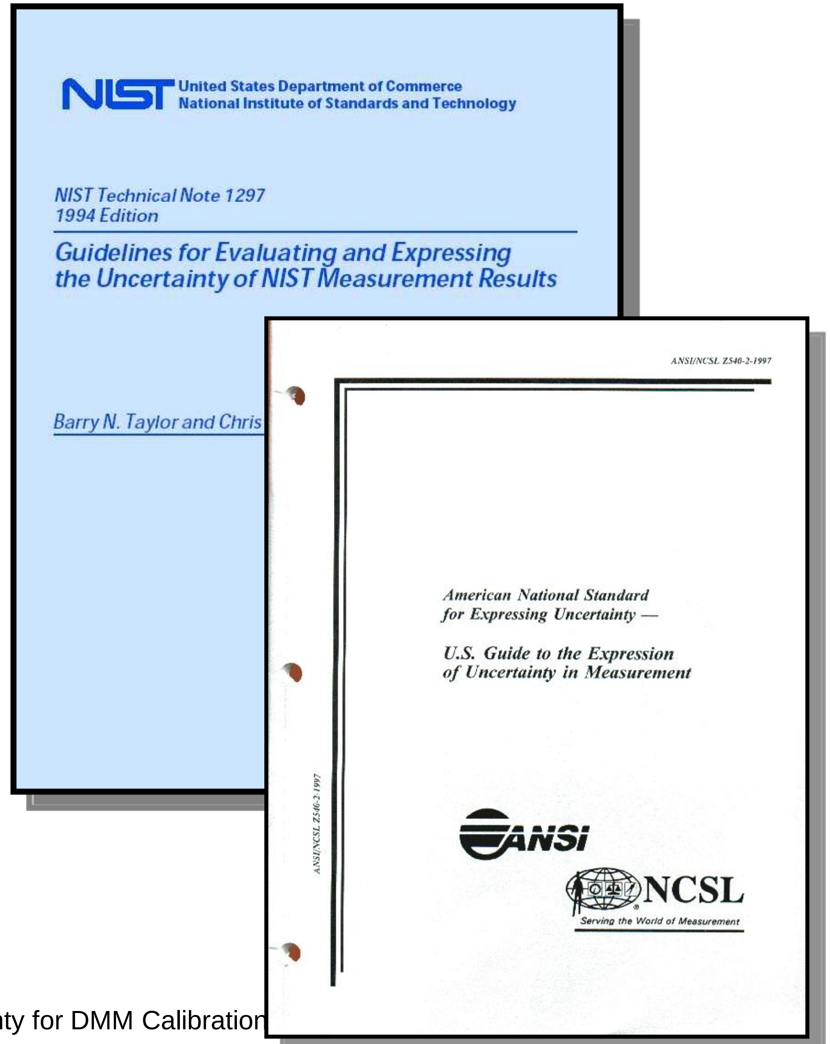
Obtain a copy of the GUMs & other references for details:

ANSI/NCSL Z540.2-1997 (R2002) U.S. Guide to Expression of Uncertainty in Measurement

http://www.ncsli.org and find it in the store under NCSLI publications

NIST Technical Note 1297

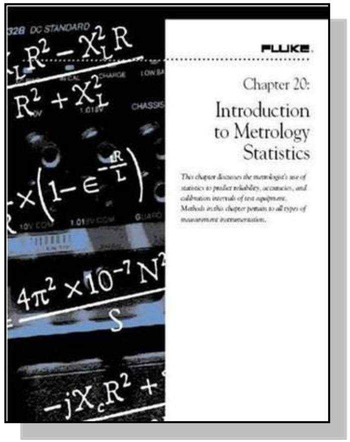
http://www.physics.nist.gov/Pubs/guidelines/contents.html



For more information (1) -

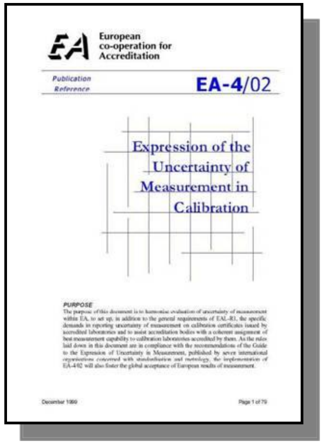
- Chapters 20-22 on Statistics & Uncertainty in the text book ***Calibration: Philosophy in Practice 2nd. Edition***
- Fluke's Training Course – Cal Lab Management for the 21st Century
- Various reference material under technical papers at the resource library on Fluke's web site:

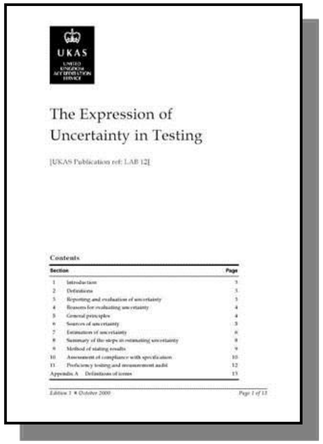
  http://www.fluke.com

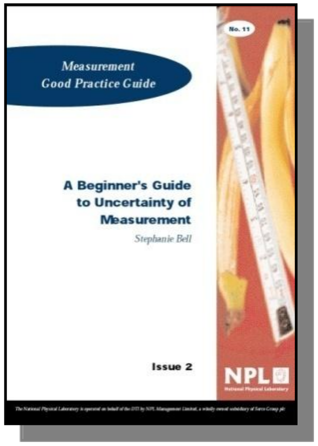


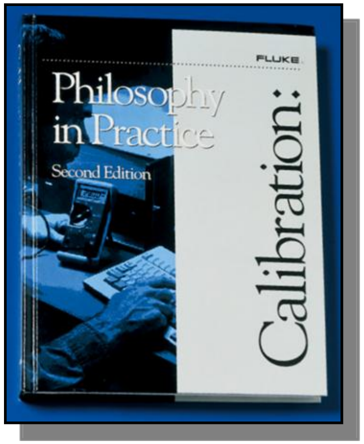
For more information (2) -

- **EA-4/02 "Expression of the Uncertainty of Measurement of Calibration"**
  http://www.european-accreditation.org
- **UKAS Publication LAB-12 "The Expression of Uncertainty In Testing"**
  http://www.ukas.com/
- **NPL UK - "A Beginner's Guide to Uncertainty of Measurement"**
  http://www.npl.co.uk/npl/
- **Fluke's "Calibration – Philosophy in Practice, Second Edition"**





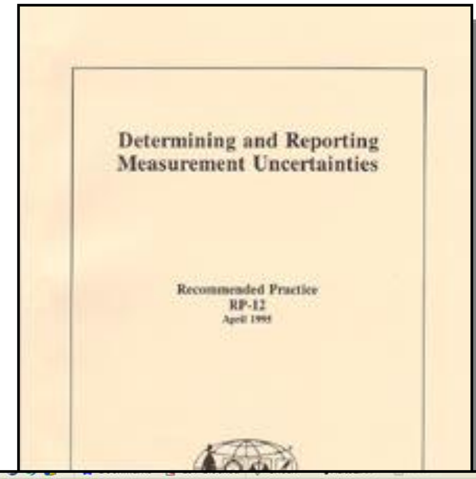


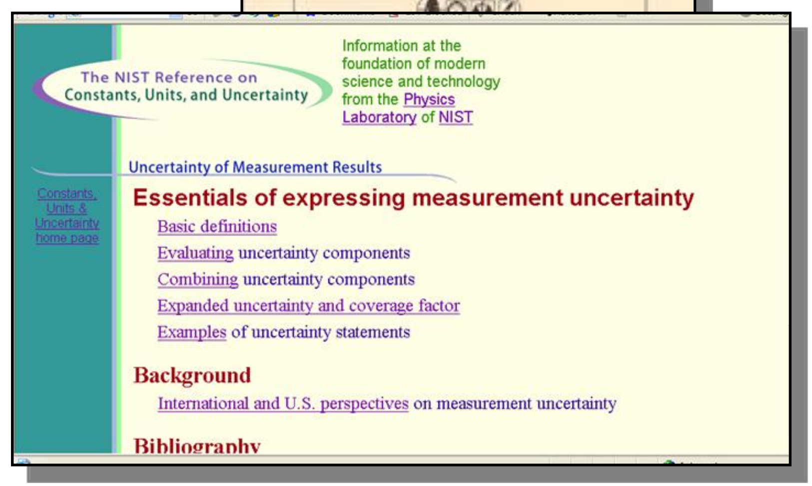


Still more references (3)

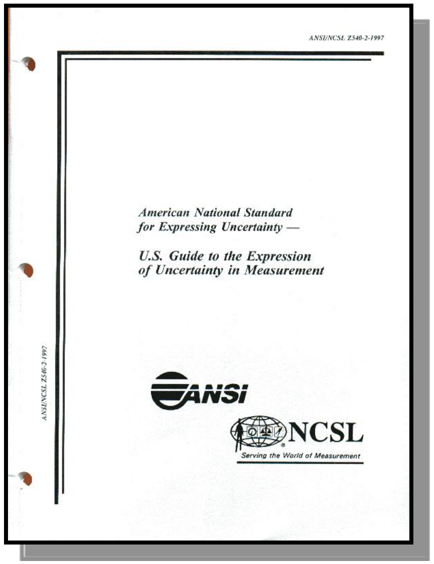
- NCSL International: RP-12 - Determining & Reporting Measurement Uncertainties
  https://www.ncsli.org/

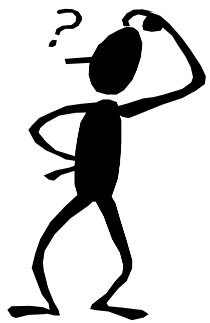
- NIST Website: Essentials of expressing measurement uncertainty
  http://physics.nist.gov/cuu/

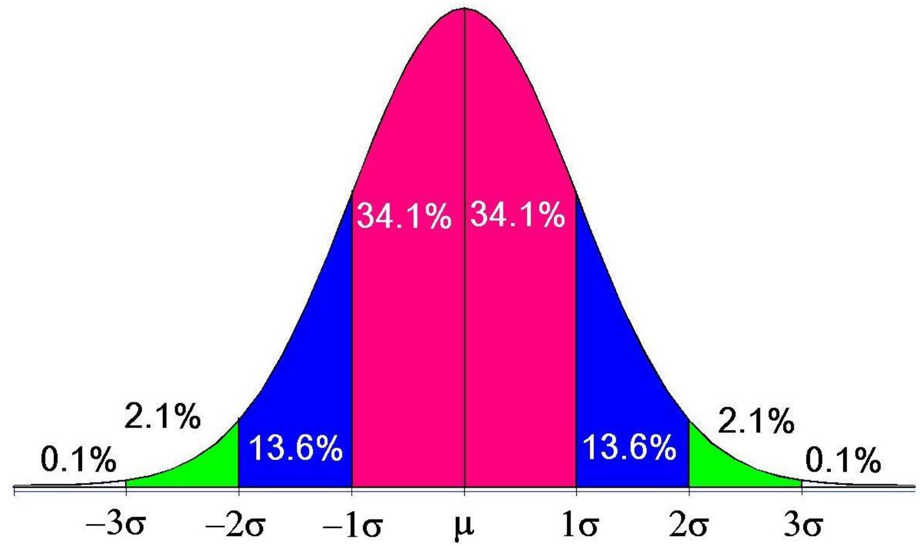


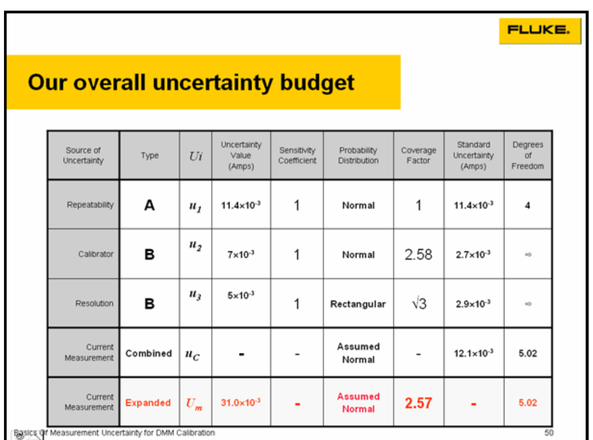


Questions?









FLUKE

Our overall uncertainty budget

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Calibrator	B	u_2	7×10^{-3}	1	Normal	2.58	2.7×10^{-3}	∞
Resolution	B	u_3	5×10^{-3}	1	Rectangular	√3	2.9×10^{-3}	∞
Current Measurement	Combined	u_C	-	-	Assumed Normal	-	12.1×10^{-3}	5.02
Current Measurement	Expanded	U_m	31.0×10^{-3}	-	Assumed Normal	2.57	-	5.02

Basics Of Measurement Uncertainty for DMM Calibration 50

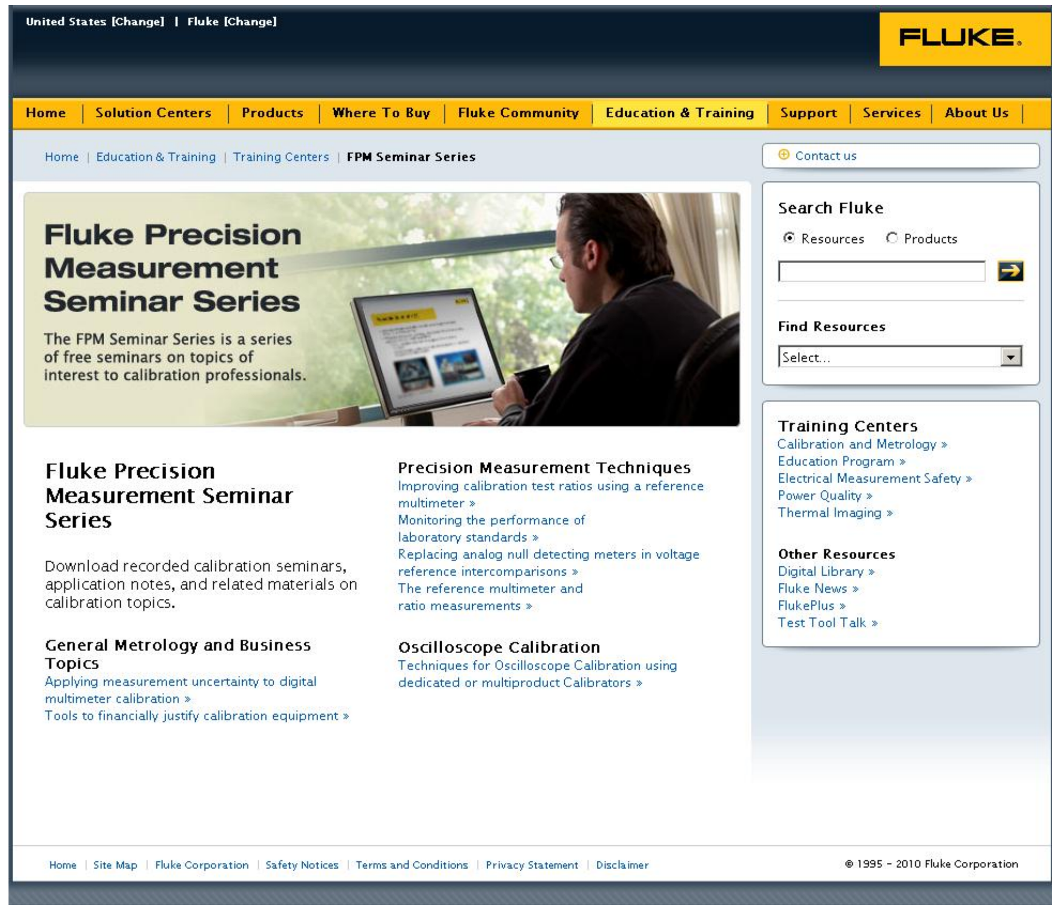
$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2}$$

Fluke Calibration Web Seminar Series

For information & reservations to attend our seminars, go to www.flukecal.com, click on the menu selection "Events & Training", and click on the "Web Seminars" selection, and again click on the desired seminar selection,

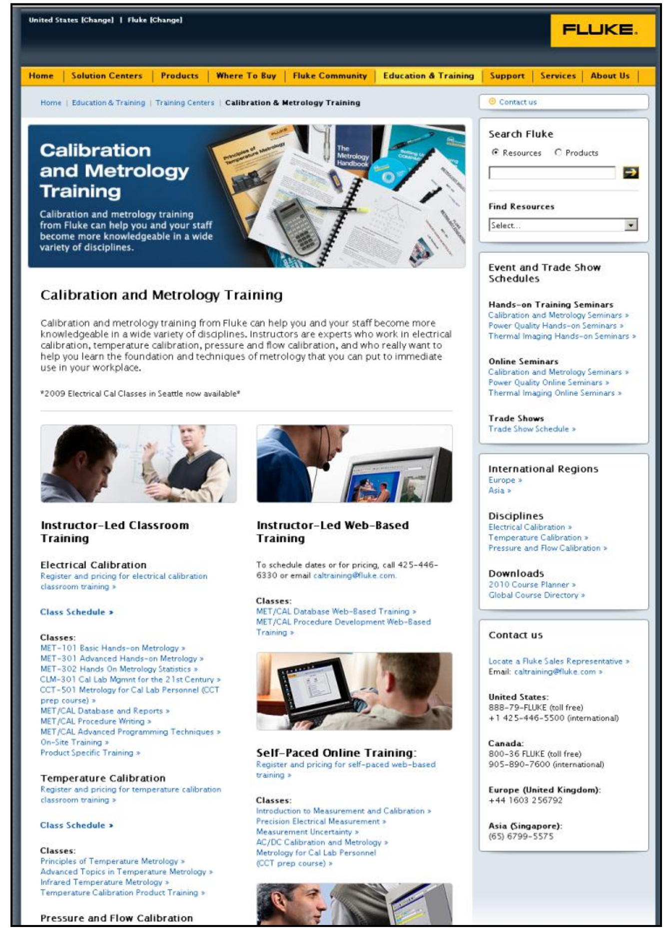
Our Seminar Topics Include:

- Precision Measurement Techniques
- Oscilloscope Calibration
- General Metrology
- Temperature Calibration
- Metrology Software
- RF Calibration



Calibration and metrology training

- **Instructor-Led Classroom Training**
  - MET-101 Basic Hands-on Metrology (new in 2007)
  - MET-301 Advanced Hands-on Metrology (new in 2007)
  - MET-302 Hands-on Metrology Statistics (new in 2009)
  - Cal Lab Management for the 21st Century
  - Metrology for Cal Lab Personnel (A CCT prep course)
  - MET/CAL Database and Reports
  - MET/CAL Procedure Writing
  - MET/CAL Advanced Programming Techniques
  - On-Site Training
  - Product Specific Training
- **Instructor-Led Web-Based Training**
  - MET/CAL Database Web-Based Training
  - MET/CAL Procedure Development Web-Based Training
- **Self-Paced Web-Based Training**
  - Introduction to Measurement and Calibration
  - Precision Electrical Measurement
  - Measurement Uncertainty
  - AC/DC Calibration and Metrology
  - Metrology for Cal Lab Personnel (A CCT prep course)
- **Self-Paced Training Tools**
  - MET/CAL-CBT7 Computer Based Training
  - MET/CAL-CBT/PW Computer-Based Training (new in 2007)
  - Cal-Book: Philosophy in Practice textbook



More information:
www.flukecal.com/training

Members of the MET/SUPPORT Gold and Priority Gold CarePlan support programs receive a 20 % discount off any Fluke calibration training course

THANK YOU !

For material related to this session, visit our web site:

http://www.fluke.com

For any questions or copies of this presentation:

email inquiries to: calibration.seminars@fluke.com